Problem Set 1 for Physical Climate System

CLIM 710

Spring 2009

Problem Set 1 – Physical Climate System

Problem 1. (taken from Hartmann, Chapter 2). Imagine a world consisting of a giant circular ribbon of super-strong material that circles a star. (Thus the diameter of the ribbon is just $2\pi r$, where r is the distance from the sun to the ribbon.) The width w of the ribbon normal to the radius vector originating at the star's center is much greater than the thickness of the ribbon in the radial direction. The width of the ribbon w is still much less than the distance of the ribbon from the star r .

If the diameter of the ribbon is the same as the diameter of Earth's orbit about the sun, and the luminosity is the same as the Earth's sun, and the albedo of the ribbon is 0.3, what is the emission (or equivalent black body) temperature of the sunlit side of the ribbon?

Consider both cases (1) heat is conducted efficiently from the sunlit to the dark side of the ribbon, and (2) no heat is conducted from the sunlit side to the dark side of the ribbon.

Problem 2. (taken from text of Hartmann chapter 3)

(2a)

- Consider the simple radiative model of the earth-atmosphere system shown in Figure 2.3 of Hartmann (in the Global Energy Balance section). Let us add another layer to the atmosphere - the 2-layer atmosphere is shown in Figure 3.10
- Again assume that all solar radiation $S_O/4\ (1-\alpha)$ is absorbed at the ground.
- Assume that the surface emits radiation at the rate of σT_s^4
- Assume that the top layer (Layer 1) emits radiation both upward and downward at the rate of σT_1^4
- Assume that the bottom layer (Layer 2) emits radiation both upward and downward at the rate of σT_2^4
- Develop equations for the energy balance at the surface, for each of the layers, and for the earth-atmosphere system as a whole.
- Prove that $T_s^4 = 3\ S_O/4\ (1-\alpha)\ /\ \sigma = 3\ T_e^4$

(2b) Derive all the energy balances for a 3-layer model, with assumptions similar to those in (2a). Prove that $T_s^4 = \quad 4\ T_e^4$

(2c) Derive all the energy balances for a 4-layer model, with assumptions similar to those in (2a). Prove that $T_s^4 = \quad 5\ T_e^4$

(2d) Generalize the result for n layers, where n can be arbitrarily large. Prove that $T_s^4 = (n+1)\ T_e^4$

Clearly this is problematic – the surface temperature will increase without bound as the number of atmospheric layers increases. *What have we not taken into account here?*

Problem 3. **From the lecture on Black Body radiation we learned that the *spectral radiance, namely the energy per unit time (or power) crossing unit area (in the normal direction) per unit solid angle and per unit frequency interval* is:**

$$B_\nu(T) = \frac{2h\nu^3}{c^2\left(e^\eta - 1\right)} \qquad \eta = \left(\frac{h\nu}{kT}\right)$$

and is known as the *Planck function*, where the meaning of all the symbols is given in the Lecture.

Estimate the value of the Planck function for a wavelength of 10μm for T = 288 K, and for a wavelength of 1 μm for T=6000 K. Take $\log_{10}(B)$ and check your answers with the graph on page 23. *Make sure to verify that all units are correct in your calculation, and show how they combine to give the correct units for the Planck function.*

Problem 4.

Prove that if B_λ is integrated over all wavelengths, we obtain the (total) black-body radiance:

$$\int_0^\infty B_\lambda(T)\, d\lambda = \int_0^\infty \frac{2hc^2}{\lambda^5\left(e^\eta - 1\right)}\, d\lambda = \frac{\sigma}{\pi} T^4$$

$$\eta = \left(\frac{hc}{k\lambda T}\right)$$

where σ is the Stefan-Boltzmann constant:

$$\sigma = \frac{2\pi^5 k^4}{15 h^3 c^2} = 5.67 \times 10^{-8}\, W m^{-2} K^{-4}$$

(Note that here k is the Boltzmann constant, and that we have not integrated over all angles). You will need the following definite integral:

$$\int_0^\infty dx \frac{x^3}{\left(e^x - 1\right)} = \frac{\pi^4}{15}$$

Problem 5.

Refer to Figure 3.10 in Hartmann showing a simple two-layer Black Body model of the atmosphere + surface, and to the the associated discussion in Section 3.8.

Consider this model, but distribute the solar heating such that $0.3\ \sigma_e T^4$ is absorbed in each of the two atmospheric layers, and the remaining $0.4\ \sigma_e T^4$ is absorbed at the surface. Calculated the new radiative equilibrium temperature for the surface, for each layer, and for the earth-atmosphere system as a whole. Explain the cause of the differences in temperature from the case shown in Figure 3.10.

Problem 6. (Radiative-Convective Equilbrium)

Refer again to Figure 3.10 in Hartmann. Place the two layers in the model at 2.5 and 5.0 km. Assume a fixed lapse rate of 6.5 K km^{-1}. Derive energy balance equations that include an additional, unknown "convective" energy flux F_2, from the surface to the lower layer (Layer 2), and a second unknown "convective" energy flux F_1 from Layer 2 to Layer 1. Solve the system assuming equilibrium: obtain the surface temperature, the two layer temperatures, and the two convective fluxes needed. (Hint: Start from the top and work down.) How do the radiative and convective fluxes compare with Figure 2.4?

Problem 7.

Refer to Figure 3.10 in Hartmann and to your results in Problem 6. Compare the pure radiative equilibrium temperatures for the two layers and surface (given in Hartmann) and the "radiative-convective" temperatures you obtained in Problem 6 with the values obtained from Figure 3.16 (From Manabe and Strickler's temperature profiles) for the appropriate levels.

Repeat Problem 6 but (a) with the top level at 7.0 km. Compare the new thermal equilibrium temperatures with those of Figure 3.16 at the appropriate levels.

Problem 8. (Layered Atmosphere with Absorption)

In the Atmospheric Radiative Transfer and Climate lecture we considered a simple one-layer atmosphere with absoprtion. Consider the two-layer generalization (simple picture showed on slide 10). In the picture, only the transmitted radiation is shown.

(a) Complete the diagram to show the absorbed radiation also. Then write the energy balance for each layer, the surface, and the entire earth-atmosphere system. Solve for T (or for σT^4) for each layer and the surface.

(b) Repeat this problem for 3 layers.

(c) Show that the surface temperature T_s obtained in parts (a) and (b) follows the general formula:

$$\sigma T_s^4 = \frac{F_0}{(2-a)}\left(2+(N-1)a\right)$$

where $F_0 = \sigma T_e^4$

Problem 8. (Layered Atmosphere with Absorption: continued)

(d) Let the asbsorption coefficient a = χ* / N, where χ * is just the optical thickness used in the model of radiative transfer derived starting on page 23 of the notes for Radiative Transfer and Climate Part II notes. Why is this a reasonable assumption? In the limit that N becomes very large, show that:

$$\sigma T_s^4 = \frac{F_0}{2}\left(2 + \chi^*\right)$$

which is just equation (38) of the Notes.

(e) Return to parts (a) and (b). Your answers for σT_1^4 and σT_2^4 should have been the same for the 2-layer and 3-layer cases. Can you prove that these quantities are independent of N, the number of layers in the model?

$\sigma T_s^4 (1-a)^2 \uparrow$ $\quad$ $\sigma T_1^4 \uparrow$ $\quad$ $\sigma T_2^4 (1-a) \uparrow$

——— (Layer 1)

$\sigma T_s^4 (1-a) \uparrow$ $\quad$ $\sigma T_1^4 \downarrow$ $\quad$ $\sigma T_2^4 \uparrow$

——— (Layer 2)

$\sigma T_s^4 \uparrow$ $\quad$ $\sigma T_e^4 \downarrow$ $\quad$ $\sigma T_1^4 (1-a) \downarrow$ $\quad$ $\sigma T_2^4 \downarrow$

——— (Surface)

Diagram of two-layer atmosphere with absorption for Problem 8

Only transmitted radiation is shown!